A

The First Maurer-Cartan Structure Relation

The first structure relation [2] defines the torsion or spin form as the exterior covariant derivative of the tetrad form:

$$T^a{}_{\mu\nu} = (D \wedge q^a)_{\mu\nu} = (d \wedge q^a)_{\mu\nu} + \omega^a{}_{\mu b} q^b{}_\nu - \omega^a{}_{\nu b} q^b{}_\mu \tag{A.1}$$

where $\omega^a{}_{\mu b}$ is the spin connection. The torsion tensor is therefore:

$$T^\lambda{}_{\mu\nu} = q^\lambda{}_a T^a{}_{\mu\nu} \tag{A.2}$$

and using the tetrad postulate:

$$T^\lambda{}_{\mu\nu} = q^\lambda{}_a \left(\partial_\mu q^a{}_\nu - \partial_\nu q^a{}_\mu + \omega^a{}_{\mu b} q^b{}_\nu - \omega^a{}_{\nu b} q^b{}_\mu \right) \tag{A.3}$$

we obtain:

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu} \,. \tag{A.4}$$

This is an expression for the torsion tensor in terms of the gamma connection of any symmetry. If the gamma connection is the symmetric Christoffel symbol:

$$\Gamma^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\nu\mu} \tag{A.5}$$

then the torsion tensor vanishes.

B

The Second Maurer-Cartan Structure Relation

The second structure relation defines the Riemann or curvature form as the exterior covariant derivative of the spin connection, regarded as a one-form:

$$R^a{}_b = D \wedge \omega^a{}_b \tag{B.1}$$

i.e

$$R^a{}_{b\nu\mu} = \partial_\nu \omega^a{}_{\mu b} - \partial_\mu \omega^a{}_{\nu b} + \omega^a{}_{\nu c}\omega^c{}_{\mu b} - \omega^a{}_{\mu c}\omega^c{}_{\nu b}. \tag{B.2}$$

It is proven in this appendix that the second structure relation is equivalent to the definition of the Riemann tensor for a gamma connection of any symmetry.

The proof starts with the tetrad postulate expressed as:

$$\omega^a{}_{\mu b} = q^a{}_\nu q^\lambda{}_b \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_b \partial_\mu q^a{}_\lambda. \tag{B.3}$$

Multiplying both sides of Eq (B.3) by $q^b{}_\lambda$ and using:

$$q^b{}_\lambda q^\lambda{}_b = 1 \tag{B.4}$$

the tetrad postulate can be expressed as:

$$\partial_\mu q^a{}_\lambda = q^a{}_\nu \Gamma^\nu{}_{\mu\lambda} - q^b{}_\lambda \omega^a{}_{\mu b}. \tag{B.5}$$

Differentiating Eq (B.3) and using the Leibnitz Theorem:

$$\begin{aligned}\partial_\nu \omega^a{}_{\mu b} =& \partial_\nu \left(q^a{}_\sigma q^\lambda{}_b \Gamma^\sigma{}_{\mu\lambda}\right) - \partial_\nu \left(q^\lambda{}_b \partial_\mu q^a{}_\lambda\right) \\ =& \partial_\nu \left(q^a{}_\sigma q^\lambda{}_b\right) \Gamma^\sigma{}_{\mu\lambda} + q^a{}_\sigma q^\lambda{}_b \partial_\nu \Gamma^\sigma{}_{\mu\lambda} \\ &- \left(\partial_\nu q^\lambda{}_b\right)\left(\partial_\mu q^a{}_\lambda\right) - q^\lambda{}_b \left(\partial_\nu \partial_\mu \left(q^a{}_\lambda\right)\right).\end{aligned} \tag{B.6}$$

Now use the Leibnitz Theorem again:

$$\partial_\nu \left(q^\lambda{}_b q^a{}_\sigma\right) = q^a{}_\sigma \partial_\nu q^\lambda{}_b + q^\lambda{}_b \partial_\nu q^a{}_\sigma \tag{B.7}$$

to obtain:

$$\begin{aligned}\partial_\nu \omega^a{}_{\mu b} = &\left(q^a{}_\sigma \Gamma^\sigma{}_{\mu\lambda} - \partial_\mu q^a{}_\lambda\right) \partial_\nu q^\lambda{}_b \\ &+ q^\lambda{}_b \Gamma^\sigma{}_{\mu\lambda}\, \partial_\nu q^a{}_\sigma + q^a{}_\sigma q^\lambda{}_b \partial_\nu \Gamma^\sigma{}_{\mu\lambda} \\ &- q^\lambda{}_b \left(\partial_\nu \partial_\mu \left(q^a{}_\lambda\right)\right)\end{aligned} \tag{B.8}$$

Now use Eq. (B.5)in Eq. (B.8):

$$\begin{aligned}\partial_\nu \omega^a{}_{\mu b} =& q^b{}_\lambda \omega^a{}_{\mu b} \partial_\nu q^\lambda{}_b + q^\lambda{}_b \Gamma^\sigma{}_{\mu\lambda}\, \partial_\nu q^a{}_\sigma \\ &+ q^\lambda{}_b q^a{}_\sigma \partial_\nu \Gamma^\sigma{}_{\mu\lambda} - q^\lambda{}_b \left(\partial_\nu \partial_\mu \left(q^a{}_\lambda\right)\right).\end{aligned} \tag{B.9}$$

Switching the μ and ν indices gives:

$$\begin{aligned}\partial_\mu \omega^a{}_{\nu b} =& q^b{}_\lambda \omega^a{}_{\nu b} \partial_\mu q^\lambda{}_b + q^\lambda{}_b \Gamma^\sigma{}_{\nu\lambda}\, \partial_\mu q^a{}_\sigma \\ &+ q^\lambda{}_b q^a{}_\sigma \partial_\mu \Gamma^\sigma{}_{\nu\lambda} - q^\lambda{}_b \left(\partial_\mu \partial_\nu \left(q^a{}_\lambda\right)\right)\end{aligned} \tag{B.10}$$

which implies:

$$\begin{aligned}\partial_\nu \omega^a{}_{\mu b} - \partial_\mu \omega^a{}_{\nu b} =& q^b{}_\lambda \left(\omega^a{}_{\mu b} \partial_\nu q^\lambda{}_b - \omega^a{}_{\nu b} \partial_\mu q^\lambda{}_b\right) \\ &+ q^b{}_\lambda \left(\Gamma^\sigma{}_{\mu\lambda}\, \partial_\nu q^a{}_\sigma - \Gamma^\sigma{}_{\nu\lambda}\, \partial_\mu q^a{}_\sigma\right) \\ &+ q^\lambda{}_b q^a{}_\sigma \left(\partial_\nu \Gamma^\sigma{}_{\mu\lambda} - \partial_\mu \Gamma^\sigma{}_{\nu\lambda}\right)\end{aligned} \tag{B.11}$$

because

$$\left(\partial_\nu \partial_\mu - \partial_\nu \partial_\mu\right) q^a{}_\lambda = 0. \tag{B.12}$$

In order to evaluate the Riemann form:

$$R^a{}_{b\nu\mu} = \partial_\nu \omega^a{}_{\mu b} - \partial_\mu \omega^a{}_{\nu b} + \omega^a{}_{\nu c} \omega^c{}_{\mu b} - \omega^a{}_{\mu c} \omega^c{}_{\nu b} \tag{B.13}$$

we need:

$$\omega^a{}_{\nu c} = q^a{}_\mu q^\lambda{}_c \Gamma^\mu{}_{\nu\lambda} - q^\lambda{}_c \partial_\nu q^a{}_\lambda \tag{B.14}$$

$$\omega^a{}_{\mu b} = q^c{}_\nu q^\lambda{}_b \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_b \partial_\mu q^c{}_\lambda \tag{B.15}$$

$$\omega^a{}_{\mu c} = q^a{}_\nu q^\lambda{}_c \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_c \partial_\mu q^a{}_\lambda \tag{B.16}$$

$$\omega^c{}_{\nu b} = q^c{}_\mu q^\lambda{}_b \Gamma^\mu{}_{\nu\lambda} - q^\lambda{}_b \partial_\nu q^c{}_\lambda. \tag{B.17}$$

It is then possible to evaluate products such as:

$$\omega^a{}_{\nu c} \omega^c{}_{\mu b} = \left(q^a{}_\mu q^\lambda{}_c \Gamma^\mu{}_{\nu\lambda} - q^\lambda{}_c \partial_\nu q^a{}_\lambda\right) \left(q^c{}_\nu q^\lambda{}_b \Gamma^\nu{}_{\mu\lambda} - q^\lambda{}_b \partial_\mu q^c{}_\lambda\right). \tag{B.18}$$

The Riemann tensor can then be evaluated using:

$$R^\sigma{}_{\lambda\nu\mu} = q^\sigma{}_a q^b{}_\lambda R^a{}_{b\nu\mu}. \tag{B.19}$$

In order to evaluate Eq (B.19) first rearrange dummy indices in Eq. (B.18) as follows:

$$\begin{aligned}&q^{\lambda}{}_{c}q^{a}{}_{\mu}q^{\lambda}{}_{b}q^{c}{}_{\nu}\Gamma^{\mu}{}_{\nu\lambda}\Gamma^{\nu}{}_{\mu\lambda}\\&\qquad\downarrow(\mu\rightarrow\sigma)\\&q^{\lambda}{}_{c}q^{a}{}_{\sigma}q^{\lambda}{}_{b}q^{c}{}_{\nu}\Gamma^{\sigma}{}_{\nu\lambda}\Gamma^{\nu}{}_{\mu\lambda}\\&\qquad\downarrow(\lambda\rightarrow\rho,\nu\rightarrow\rho)\\&q^{\rho}{}_{c}q^{a}{}_{\sigma}q^{\lambda}{}_{b}q^{c}{}_{\rho}\Gamma^{\sigma}{}_{\nu\rho}\Gamma^{\rho}{}_{\mu\lambda}=q^{a}{}_{\sigma}q^{\lambda}{}_{b}\Gamma^{\sigma}{}_{\nu\rho}\Gamma^{\rho}{}_{\mu\lambda}\end{aligned}\tag{B.20}$$

Secondly cancel the term $q^{\lambda}{}_{b}\Gamma^{\sigma}{}_{\nu\rho}\,\partial_{\nu}q^{a}{}_{\sigma}$ in Eq. (B.11) with the term $-(q^{\lambda}{}_{c}\partial_{\nu}q^{a}{}_{\lambda})$ $(q^{\lambda}{}_{b}q^{c}{}_{\nu}\Gamma^{\nu}{}_{\mu\lambda})$ In Eq. (B.18) by rearranging dummy indices as follows:

$$\begin{aligned}&-q^{\lambda}{}_{c}q^{\lambda}{}_{b}q^{c}{}_{\nu}\Gamma^{\nu}{}_{\mu\lambda}\,\partial_{\nu}q^{a}{}_{\lambda}\\&\qquad\downarrow(\lambda\rightarrow\sigma)\\&-q^{\sigma}{}_{c}q^{\lambda}{}_{b}q^{c}{}_{\nu}\Gamma^{\nu}{}_{\mu\lambda}\,\partial_{\nu}q^{a}{}_{\sigma}\\&\qquad\downarrow(\nu\rightarrow\sigma)\\&-q^{\sigma}{}_{c}q^{\lambda}{}_{b}q^{c}{}_{\sigma}\Gamma^{\sigma}{}_{\mu\lambda}\,\partial_{\nu}q^{a}{}_{\sigma}=-q^{\lambda}{}_{b}\Gamma^{\sigma}{}_{\mu\lambda}\,\partial_{\nu}q^{a}{}_{\sigma}.\end{aligned}\tag{B.21}$$

Finally cancel the term $-q^{b}{}_{\lambda}\omega^{a}{}_{\nu b}\partial_{\mu}q^{\lambda}{}_{b}$ in Eq. (B.11) with the term $q^{\lambda}{}_{c}q^{\lambda}{}_{b}$ $(\partial_{\nu}q^{a}{}_{\lambda})(\partial_{\mu}q^{c}{}_{\lambda})-q^{a}{}_{\mu}q^{\lambda}{}_{c}q^{\lambda}{}_{b}\Gamma^{\mu}{}_{\nu\lambda}\,\partial_{\mu}q^{c}{}_{\lambda}$ in Eq. (B.18). To do this rewrite the Eq (B.18) term as $q^{\lambda}{}_{c}q^{\lambda}{}_{b}\partial_{\mu}q^{c}{}_{\lambda}(\partial_{\nu}q^{a}{}_{\lambda}-q^{a}{}_{\mu}\Gamma^{\mu}{}_{\nu\lambda})$ and use the tetrad postulate:

$$\partial_{\nu}q^{a}{}_{\lambda}=q^{a}{}_{\mu}\Gamma^{\mu}{}_{\nu\lambda}-q^{b}{}_{\lambda}\omega^{a}{}_{\nu b}\tag{B.22}$$

to obtain:

$$-q^{\lambda}{}_{c}q^{\lambda}{}_{b}q^{b}{}_{\lambda}\omega^{a}{}_{\nu b}\partial_{\mu}q^{c}{}_{\lambda}=-q^{c}{}_{\lambda}\omega^{a}{}_{\nu b}\partial_{\mu}q^{c}{}_{\lambda}.\tag{B.23}$$

We therefore obtain:

$$-q^{b}{}_{\lambda}\omega^{a}{}_{\nu b}\partial_{\mu}q^{\lambda}{}_{b}-\left(-q^{\lambda}{}_{c}\omega^{a}{}_{\nu b}\partial_{\mu}q^{c}{}_{\lambda}\right)=-\omega^{a}{}_{\nu b}\left(q^{c}{}_{\lambda}\partial_{\mu}q^{\lambda}{}_{c}+q^{\lambda}{}_{c}\partial_{\mu}q^{c}{}_{\lambda}\right).\tag{B.24}$$

In order to show that this is zero use:

$$q^{\lambda}{}_{c}q^{c}{}_{\lambda}=1\tag{B.25}$$

and differentiate:

$$\partial_{\mu}(q^{\lambda}{}_{c}q^{c}{}_{\lambda})=0.\tag{B.26}$$

Finally use the Leibnitz Theorem to obtain:

$$q^{\lambda}{}_{c}\partial_{\mu}q^{c}{}_{\lambda}+q^{c}{}_{\lambda}\partial_{\mu}q^{\lambda}{}_{c}=0.\tag{B.27}$$

The remaining terms give the Riemann tensor for any gamma connection:

$$R^{\lambda}{}_{\sigma\nu\mu}=\partial_{\nu}\Gamma^{\sigma}{}_{\mu\lambda}-\partial_{\mu}\Gamma^{\sigma}{}_{\nu\lambda}+\Gamma^{\sigma}{}_{\nu\rho}\Gamma^{\rho}{}_{\mu\lambda}-\Gamma^{\sigma}{}_{\mu\rho}\Gamma^{\rho}{}_{\nu\lambda}\tag{B.28}$$

quod erat demonstrandum.

C

The First Bianchi Identity

The first Bianchi identity of differential geometry is a balance of spin and curvature

$$D \wedge T^a := R^a{}_b \wedge q^b \tag{C.1}$$

and becomes the homogeneous field equation of the Evans unified field theory:

$$D \wedge F^a := R^a{}_b \wedge A^b \tag{C.2}$$

using the Evans Ansatz:

$$A^a = A^{(0)} q^a \tag{C.3}$$

So it is important to thoroughly understand the structure and meaning of the first Bianchi identity as in this Appendix. In order to proceed we need the following general definitions [2] of the exterior derivative and wedge product for any differential form:

$$(d \wedge A)_{\mu_1 \cdots \mu_{p+1}} = (p+1)\, \partial_{[\mu_1} A_{\mu_2 \cdots \mu_{p+1}]} \tag{C.4}$$

$$(A \wedge B)_{\mu_1 \cdots \mu_{p+q}} = \frac{(p+q)!}{p!q!}\, (p+1)\, A_{[\mu_1 \cdots \mu_p} B_{\mu_{p+1} \cdots \mu_{p+q}]}. \tag{C.5}$$

Eq (C.4) defines the exterior derivative of a p-form and Eq. (C.5) defines the wedge product of a p-form and a q-form. We also use the fact that the spin connection is a one-form [2]. The exterior covariant derivative of a one-form $X^a{}_\mu$, for example, then follows as:

$$(D \wedge X)^a{}_{\mu\nu} = (d \wedge X)^a{}_{\mu\nu} + (\omega \wedge X)^a{}_{\mu\nu} \tag{C.6}$$

where:

$$(d \wedge X)^a{}_{\mu\nu} = \partial_\mu X^a{}_\nu - \partial_\nu X^a{}_\mu \tag{C.7}$$

$$(\omega \wedge X)^a{}_{\mu\nu} = \omega^a{}_{\mu b} X^b{}_\nu - \omega^a{}_{\nu b} X^b{}_\mu . \tag{C.8}$$

Eqs. (C.7) and (C.8) follow using:

$$p = 1, q = 1, \mu_1 = \mu, \mu_2 = \nu \tag{C.9}$$

and

$$(d \wedge A)_{\mu_1 \mu_2} = (d \wedge A)_{\mu\nu} = 2\partial_{[\mu} A_{\nu]} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{C.10}$$

$$(A \wedge B)_{\mu_1 \cdots \mu_{p+q}} = (A \wedge B)_{\mu\nu} = \frac{2!}{1!1!} A_{[\mu} B_{\nu]} = A_\mu B_\nu - A_\nu B_\mu \tag{C.11}$$

Now extend this method to the exterior covariant derivative of a two-form, using:

$$\begin{aligned}(d \wedge A)_{\mu_1 \mu_2 \mu_3} &= 3\partial_{[\mu_1} A_{\mu_2 \mu_3]} \\ &= \partial_\mu A_{\nu\rho} + \partial_\nu A_{\rho\mu} + \partial_\rho A_{\mu\nu}\end{aligned} \tag{C.12}$$

and

$$\begin{aligned}(A \wedge B)_{\mu_1 \mu_2 \mu_3} &= \frac{3!}{2!1!} A_{[\mu_1} B_{\mu_2 \mu_3]} = 3A_{[\mu} B_{\nu\rho]} \\ &= A_\mu B_{\nu\rho} + A_\nu B_{\rho\mu} + A_\rho B_{\mu\nu}\end{aligned} \tag{C.13}$$

Therefore the exterior covariant derivative of the torsion or spin form used in the first Bianchi identity is:

$$(D \wedge T)^a{}_{\mu\nu\rho} = (d \wedge T)^a{}_{\mu\nu\rho} + (\omega \wedge T)^a{}_{\mu\nu\rho} \tag{C.14}$$

where:

$$(d \wedge T)^a{}_{\mu\nu\rho} = \partial_\mu T^a{}_{\nu\rho} + \partial_\nu T^a{}_{\rho\mu} + \partial_\rho T^a{}_{\mu\nu} \tag{C.15}$$

$$(\omega \wedge T)^a{}_{\mu\nu\rho} = \omega^a{}_{\mu b} T^b{}_{\nu\rho} + \omega^a{}_{\nu b} T^b{}_{\rho\mu} + \omega^a{}_{\rho b} T^b{}_{\mu\nu} \tag{C.16}$$

and where:

$$T^a{}_{\mu\nu} = \left(\Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}\right) q^a{}_\lambda \tag{C.17}$$

Similarly:

$$\begin{aligned}R^a{}_b \wedge q^b &= R^a{}_{b\mu\nu} q^b{}_\rho + R^a{}_{b\nu\rho} q^b{}_\mu + R^a{}_{b\rho\mu} q^b{}_\nu \\ &= R^a{}_{\mu\nu\rho} + R^a{}_{\nu\rho\mu} + R^a{}_{\rho\mu\nu} \\ &= \left(R^\sigma{}_{\mu\nu\rho} + R^\sigma{}_{\nu\rho\mu} + R^\sigma{}_{\rho\mu\nu}\right) q^a{}_\sigma .\end{aligned} \tag{C.18}$$

So the first Bianchi identity becomes:

$$\partial_\mu T^a{}_{\nu\rho} + \omega^a{}_{\mu b} T^b{}_{\nu\rho} + \cdots = R^\sigma{}_{\mu\nu\rho} q^a{}_\sigma + \cdots \quad . \tag{C.19}$$

Using Eq. (C.17), Eq. (C.19) becomes:

$$\begin{aligned}\partial_\mu \left(\left(\Gamma^\lambda{}_{\nu\rho} - \Gamma^\lambda{}_{\rho\nu}\right) q^a{}_\lambda \right) + \omega^\lambda{}_{\mu b} \left(\Gamma^\lambda{}_{\nu\rho} - \Gamma^\lambda{}_{\rho\nu}\right) q^b{}_\lambda + \cdots \\ = R^\lambda{}_{\mu\nu\rho} q^a{}_\lambda + \cdots\end{aligned} \tag{C.20}$$

Using the Leibnitz Theorem Eq. (C.20) becomes:

$$\left(\partial_\mu \Gamma^\lambda_{\ \nu\rho} - \partial_\mu \Gamma^\lambda_{\ \rho\nu}\right) q^a_{\ \lambda} + \left(\partial_\mu q^a_{\ \lambda} + \omega^a_{\ \mu b} q^b_{\ \lambda}\right) \left(\Gamma^\lambda_{\ \nu\rho} - \Gamma^\lambda_{\ \rho\nu}\right) \\ + \cdots \qquad = R^\lambda_{\ \mu\nu\rho} q^a_{\ \lambda} + \cdots \tag{C.21}$$

Now use the tetrad postulate:

$$\partial_\mu q^a_{\ \rho} + \omega^a_{\ \mu b} q^b_{\ \sigma} = \Gamma^\lambda_{\ \mu\sigma} q^a_{\ \lambda} \tag{C.22}$$

in Eq. (C.21) to obtain:

$$\begin{aligned} &\partial_\mu \Gamma^\lambda_{\ \nu\rho} - \partial_\nu \Gamma^\lambda_{\ \mu\rho} + \Gamma^\lambda_{\ \mu\sigma} \Gamma^\sigma_{\ \nu\rho} - \Gamma^\lambda_{\ \nu\sigma} \Gamma^\sigma_{\ \mu\rho} \\ &+ \partial_\nu \Gamma^\lambda_{\ \rho\mu} - \partial_\rho \Gamma^\lambda_{\ \nu\mu} + \Gamma^\lambda_{\ \nu\sigma} \Gamma^\sigma_{\ \rho\mu} - \Gamma^\lambda_{\ \rho\sigma} \Gamma^\sigma_{\ \nu\mu} \\ &+ \partial_\rho \Gamma^\lambda_{\ \mu\nu} - \partial_\mu \Gamma^\lambda_{\ \rho\nu} + \Gamma^\lambda_{\ \rho\sigma} \Gamma^\sigma_{\ \mu\nu} - \Gamma^\lambda_{\ \mu\sigma} \Gamma^\sigma_{\ \rho\nu} \\ &:= R^\lambda_{\ \rho\mu\nu} + R^\lambda_{\ \mu\nu\rho} + R^\lambda_{\ \nu\rho\mu}. \end{aligned} \tag{C.23}$$

The Riemann tensor for any connection (Appendix two) is:

$$R^\lambda_{\ \rho\mu\nu} = \partial_\mu \Gamma^\lambda_{\ \nu\rho} - \partial_\nu \Gamma^\lambda_{\ \mu\rho} + \Gamma^\lambda_{\ \mu\sigma} \Gamma^\sigma_{\ \nu\rho} - \Gamma^\lambda_{\ \nu\sigma} \Gamma^\sigma_{\ \mu\rho}\,, \tag{C.24}$$

and so Eq. (C.23) is an identity made up of the cyclic sum of three Riemann tensors on either side. The familiar Bianchi identity of the famous Einstein gravitational theory is the SPECIAL CASE when the cyclic sum vanishes:

$$R^\lambda_{\ \rho\mu\nu} + R^\lambda_{\ \mu\nu\rho} + R^\lambda_{\ \nu\rho\mu} = 0. \tag{C.25}$$

Eq. (17.25) is true if and only if the gamma connection is the symmetric Christoffel symbol:

$$\Gamma^\lambda_{\ \mu\nu} = \Gamma^\lambda_{\ \nu\mu}\,. \tag{C.26}$$

It is not at all clear using tensor notation (Eq. (17.23)) that the first Bianchi identity is a balance of spin and curvature. In order to see this we need the differential form notation of Eq. (C.1) and this is of key importance for the development of the Evans unified field theory.